On the relationship of tornado path length and width to intensity

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Abstract

Reported path length and widths of tornadoes have been modelled using Weibull distributions for different F-scale values. The fits are good over a wide range of lengths and widths. Path length and width tend to increase with increasing F-scale, although the temporal nonstationarity of the data for some parts of the data (such as width of F3 tornadoes) is large enough that caution must be exercised in interpretation of short periods of record. The statistical distributions also demonstrate that, as the length or width increases, the most likely F-scale value associated with the length or width tends to increase. Nevertheless, even for long or wide tornadoes, there is a significant probability of a range of possible F-values, so that simple observation of the length or width is insufficient to make an accurate estimate of the F-scale.

1. Introduction

The relationship of the size of tornadoes to the intensity of their damage is of importance in modelling the hazards associated with tornadoes (Schaefer et al. 1985, Meyer et al. 2002, Schaefer et al. 2002) and potentially could be useful in attempting to forecast intensity, if the relationships are found to be strong enough. Tecson et al. (1979) and Abbey and Fujita (1979) described early efforts to look at the width and length of reported tornadoes in association with intensity, as measured by the Fujita scale, or F-scale (Fujita 1981). McCarthy (2003) recently has updated that work and shown that width of tornado paths tends to increase with the intensity, but there is considerable overlap between classes.

If the distributions of width and length can be modelled with simple theoretical probability distributions, then the information associated with the thousands of observed tornado path length and widths could be summarized in a two or three numbers. Development of statistically-based hazard models for tornadoes could then use the parameters of the distributions as input. Changes in the parameterized distributions with time or space and differences between different classifications could also be investigated.

Here, I will describe an attempt to model the distribution of path length and widths using Weibull distributions. The relationship of path length and width to damage classification will be investigated, and an estimate of the probability of a particular F-scale, given the length or width of a tornado will be developed. Finally, the temporal robustness of the estimates and implications for interpretation of the data will be discussed.

2. Data and methodology

The dataset consists of all tornadoes in the National Weather Service (NWS) Storm Prediction Center (SPC) database of tornadoes in the United States from 1950-2001. Schaefer and Edwards (1999) and McCarthy (2003) described the database and the changes that have occurred over the years. The database is available on-line from the SPC website (http://www.spc.noaa.gov). Path lengths are reported in miles and widths in yards[1] and the maximum damage as rated by the F-scale. Doswell and Burgess (1988) have discussed problems with assigning F-scale ratings, but it is hoped that the relatively large sample size here will overcome random errors in assignments. If systematic biases exist, they are extremely difficult to detect and there is nothing that can be done about them. Only those events with a reported length or width are included in the analysis. Over 40,000 tornadoes are included.

Weibull distributions have been fit to the observed path length and width data. Weibull distributions are appropriate for the problem because they are nonnegative and have positive skewness. They have been used historically to model wind speed distributions (Wilks 1995). The probability distribution function for a Weibull distribution is given by

[1] The mean width was reported prior to and including 1994 and the maximum width after 1994.

$$f(x) = \left(\frac{\alpha}{\beta}\right)\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], \quad (1)$$

where α, and β are the two parameters that define the distribution and x, α, and β are greater than zero. α is the shape parameter. If $\alpha \leq 1$, the distribution is shaped like a reverse "J", with maximum values of the function as $x \rightarrow 0$. For $\alpha > 1$, the function peaks at greater values of x. If $\alpha = 1$, the distribution reduces to the exponential distribution. β is a scale parameter, stretching or compressing the distribution along the x-axis for any particular value of α. The parameters have been estimated with a maximum likelihood technique by using software included in the Department of Defense's Composite Materials Handbook (Department of Defense 2002). The mean of the Weibull distribution, μ, is given by

$$\mu = \beta\Gamma\left(1 + \frac{1}{\alpha}\right) \quad (2)$$

where $\Gamma()$ is the gamma function. In the work here, the argument of the $\Gamma()$ ranges from approximately 0.6 to 1.1 for most calculations. As a result, typically, $0.9\beta < \mu < 1.5\beta$, so that the mean width and length are on the order of β.

The distributions have been fit over a variety of time intervals. Sample-size limitations make interpretation of some of those time intervals for the highest values of the F-scale difficult. For example, there are only 51 F5[2] tornadoes in the record, so that interpretation of the distributions on anything less than the complete record for F5 tornadoes is questionable. On the other hand, there are approximately 7500 F2 tornadoes

[2] "Fn" implies that the tornado was rated as "n" on the F-scale, regardless of its true meteorological intensity.

with path length and width information, so that short time ranges provide an adequate sample size.

3. Results

a. The complete record

Quantile-quantile (q-q) plots are a useful tool to assess the goodness of fit of the models qualitatively (Wilks 1995). Quantile-quantile plots compare the values associated with the same quantile of an empirical (observed) and modelled probability distribution function. For example, a plot of the 10^{th}, 20^{th}, 30^{th}, etc. percentiles of the empirical distribution versus the 10^{th}, 20^{th}, 30^{th}, etc. percentiles of the modelled distribution shows how well the data are fit by the model. In this case, the quantiles are plotted for all values that the empirical quantiles exist. The "quantization" of the reports is a limit on how these plots can be made. As an example, the F1 length q-q plot begins with the 22^{nd} percentile, since 22% of the observations have path lengths of 0.1 mile (0.16 km). The gaps in the plots are a result of gaps in the observational record.

The fits to the length data improve with increasing F-scale. The fits are not especially good at F0 and, to a lesser extent, F1, although they are better than fits assuming a normal distribution for all values. In general, all of the modelled distributions overestimate the empirical distribution for short lengths, as indicated by the points being below the diagonal, perfect-fit line (Fig. 1). The problems at short lengths are a result of the quanitization of the observations. The F0 tornado fit departs from the observations at about 4 km, underestimating the distribution for long lengths (Fig. 1a), but it is important to note that this is at the 93rd percentile, so that the errors are for rare events. The underestimation of density at long path lengths decreases for F1 and F2 tornadoes and is very small by F3 tornadoes. The picture is very similar for the widths (Fig. 2). Fits improve with increasing width and the statistical distribution tends to overestimate the density at low widths.

The calculated mean path length for tornadoes increases from a little over 1 km for F0 tornadoes to over 50 km for F5 tornadoes (Table 1). The mean length roughly doubles with each value of the F-scale from F1 to F4. The change from F0 to F1 is somewhat larger and there is less change from F4 to F5. The shape of the distribution changes as well, with the α parameter increasing, in general, with increasing F-scale. This implies that the maximum probability moves away from zero length for the high F values. The distributions for the different F-scale values are well separated except for perhaps between F4 and F5, as illustrated by the cumulative distribution functions for the various values of F (Fig. 3.)

The mean width also increases with F-scale (Table 2), from less than 30 m for F0 to more than 550 m for F5. It doubles approximately with each F value from F0 to F4, with only a slight increase from F4 to F5. There is less consistency in the change in the α parameter than in the case of path length. Only the F5 tornadoes have α greater than 1. The distributions are less distinct for the high F-values, particularly at wide widths (Fig. 4.)

The distributions represent the probability of a path length (width) given that a tornado of a particular intensity occurs. It is easy to invert the problem to calculate the probability of a particular intensity given that a path length (width) is observed. To do so, all that must be done is to weight the distributions given by (1) by the number of tornadoes at each F-scale and then divide the result for each F-scale by the sum of the weighted distributions at any given length (width). The results illustrate the potential utility and pitfalls of estimating a damage rating simply from the length or width data associated with a tornado.

At short path lengths (<5 km), the most probable F-scale is F0 (Fig. 5.) Over a broad range (~30-170 km), the most likely F-scale is F2. Over that range, however, there is a significant probability of at least 5% for any value between F1 and F4. Length information appears to be potentially of some limited utility for setting lower bounds on the likely intensity of tornadoes. Less than 5% of tornadoes with path lengths of 25 km are rated F0. The 5% cut-off for all F0 or F1 tornadoes is approximately 160 km. Given the rarity of tornadoes of that length (the most recent one in the record was in 1992), this is not a strong result in a practical sense. It is also of interest to note that, because of the rarity of F5 tornadoes, they are never the most likely event at any length.

Width information potentially has more value in putting limits on the likely intensity rating (Fig. 6). Less than 20% of tornadoes with widths of 400 m and less than 10% of tornadoes with widths of 500 m are rated F0 or F1. Thus, one can have some confidence that a wide tornado is probably at least F2. Finer distinctions based on width are difficult to justify. From 500 m to 1500 m, the distributions suggest that there is at least a 10% probability that a tornado will be F2 or F3 or F4. Again, because of the rarity of F5 tornadoes, they are never the most likely event at any width.

b. 4-year periods

Consideration of shorter time periods in the record allows for the opportunity to look at the stability of the estimates of the parameters in time and to put confidence limits on the long-term values. The typical number of reports per year varies dramatically through time, as illustrated by the number of reports in overlapping four-year periods (Fig. 7)[3]. The rapid increase at the beginning of the period of record is likely a result of the beginning of real-time forecasting of tornadoes by the NWS, with the beginning of the Severe Weather Unit of the Weather Bureau in 1952 through the move of the unit from Washington to Kansas City in 1954 (Corfidi 1999). The number of weak tornadoes (F0 and F1) has increased dramatically in the dataset, with the F0 increasing particularly when data from the year 1990 are first included. Part of this increase for F0 tornadoes is a result of a policy change in 1982 to assign F0 to all tornadoes that did not have rated damage (D. McCarthy, 2003 personal communication). An additional component of the increase is likely due to increasing population in the western US and better public awareness. The numbers of F2 and stronger tornadoes show a slightly downward trend, with an apparent step function decrease in the mid-1970s. The step function may be a result of the beginning of real-time damage surveys when the Fujita scale was adopted by the NWS. Almost all 4-year periods have at least 100 F3 tornadoes and 20 F4 tornadoes in them. Thus, the sample size allows some hope of estimating the parameters of the distribution from a 4-year period.

[3] F5 tornado reports are not shown because of their small number. There are no more than 11 in any 4-year period and, in most periods, there are no more than 5. Clearly, estimates of the statistical distribution, based on such a small sample, would have little meaning.

In order to get a handle on variability, I have randomly selected four years out of the record, calculated the parameters from those years, and then repeated that process 1000 times to compute distributions of the parameters. Box and whisker plots illustrate the variability of the mean values (Figs. 8 and 9). Random collections of years clearly show a distinction between the different F-values with more damaging tornadoes being longer and wider.

The estimates are not consistently robust in time, however. Most, such as the mean length of F3 tornadoes are quite consistent in time (Fig. 10). Others, particularly associated with the weaker tornadoes, show a decrease in the early periods, as illustrated by the mean width of F0 tornadoes (Fig. 11). This could be explained by the greater ease of detecting the effects of only the largest of all weak tornadoes. Detection efficiency of weak tornadoes is always a problem in climatological studies and it may be that the larger ones are more likely to be detected. As a result, in the period of poorer detection, only the largest weak tornadoes make the record, biasing the results upward.

Perhaps the most puzzling record is the widths of F3 tornadoes, which shows a slow increase beginning in early 1970s, when the mean reported path width was a little less than 200 m, to the mid-1990s, when the mean reported width was over 500 m, followed by a rapid decrease in the late 1990s and early 2000s (Fig. 12). Both the minimum and the maximum 4-year periods are outside of the 95% confidence intervals for the long-term mean. The peak estimate for the width is higher than any of the random combinations that went into estimates in Fig. 9, which peaked at 467 m. It fits well in the overall distribution of widths for F4 tornadoes, so that mid-1990s F3 tornadoes had reported widths comparable to the long-term record of F4 tornadoes. Although there was a policy change in the NWS to report maximum path width, instead of mean path width, that did not occur until 1994 (McCarthy 2003). Thus, the width increase takes place for a long period of time before the policy change, and a decrease follows not long after the policy change. It is not easy to explain why the mean width would increase so much. Maximum width might be expected to increase with greater concentration on damage surveys, if a greater fraction of the path is surveyed. It is not obvious that the effect on the mean path should be very large and, it is not obvious at all why it should occur with the F3 tornadoes and not with others. It may be nothing more than coincidence, but it suggests that caution should be applied to statistics from short periods of record.

4. Concluding remarks

Weibull distributions provide reasonably good fits to the record of reported tornado path lengths and widths. The goodness of the fits implies that Weibull distributions may be useful in developing statistically-based models of tornado hazards, especially given the fact that a closed form exists for the cumulative distribution function, making computations with them very efficient if a large number of tornadoes are being simulated. For any comparable period of record, lengths and widths in general increase with increasing damage scale. Physically, this seems plausible since a longer path length means that there is more opportunity for damage to occur and, in general, longer path lengths are associated with longer-lived tornadoes and/or faster storm motions. Assuming the tornado is nearly symmetric, wider tornadoes will take longer to pass over a particular point, giving more time for damage to occur even if peak wind speeds are the same. There is sufficient nonstationarity in the time series, however, that comparison of different periods of record could lead to confusion, so that caution must be exercised in producing parameters. As an example, F3 tornado widths are far from stationary. The timing of changes in reported parameters such as the path width don't match up particularly well with the timing of policy changes, leaving open the question of the true effects of those policy changes. It is possible that subtle "unofficial" changes in the way that damage path information is collected might have an impact on the statistics.

From inverting the probability distributions for size versus F-scale to give a probability of F-scale given size, it's clear that, in general, the most likely F-scale increases with increasing reported tornado size, although F5 tornadoes are never the most likely event for any size. It's also clear, though, that for most length and widths, there is a wide range of F-scale values associated with significant probability of occurrence. Since it's possible to get an estimate of the width in some cases from observing tornadoes in real time, it may be possible to guess that it is highly likely that a wide tornado (say, 500 m) is at least of F2 intensity, but going much beyond that statement is questionable. Thus, the use of length and width in forecasting intensity is of limited value.

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F	N	α	β	μ
0	16441	0.65	0.9	1.4
1	14450	0.62	3.1	4.7
2	7503	0.64	7.5	10.7
3	1967	0.83	20.4	22.5
4	469	1.01	43.9	43.6
5	51	1.20	57.7	54.6

Table 1: Parameters of the Weibull distributions for fits to length data for 1950-2001. F is the F-scale, N is the number of tornadoes in each class, α and β are the parameters of the distribution, and μ is the mean of the distribution. β and μ are in km.

F	N	α	β	μ
0	16166	0.94	27.2	28.4
1	14383	0.85	57.7	64.0
2	7487	0.79	107.4	125.9
3	1960	0.84	240.2	263.6
4	469	1.00	461.2	460.7
5	51	1.71	620.5	555.5

Table 2: Same as Table 1, except for width data in m.

Figure Captions

Fig. 1: Quantile-quantile plots for Weibull distribution fits to path length data for tornadoes of various F scales from 1950-2001. Note logarithmic axes. Diagonal line indicates perfect fit.

Fig. 2: Same as Fig. 1, except for path width.

Fig. 3: Cumulative distribution functions for path length of tornadoes by F-scale. Thin lines are F0-F2; thick lines are F3-F5, with solid, gray, and dashed lines representing the three F-scale values in each group, respectively.

Fig. 4: Same as Fig. 3 except for path width.

Fig. 5: Probability (p) of F-scale value, given path length. Convention for lines as in Fig. 3.

Fig. 6: Same as Fig. 5, except for path width.

Fig. 7: Number of tornadoes in overlapping 4-year periods by F-scale. a) F0 (black solid), F1 (gray solid), F2 (dashed). b) F3 (black solid), F4 (gray solid).

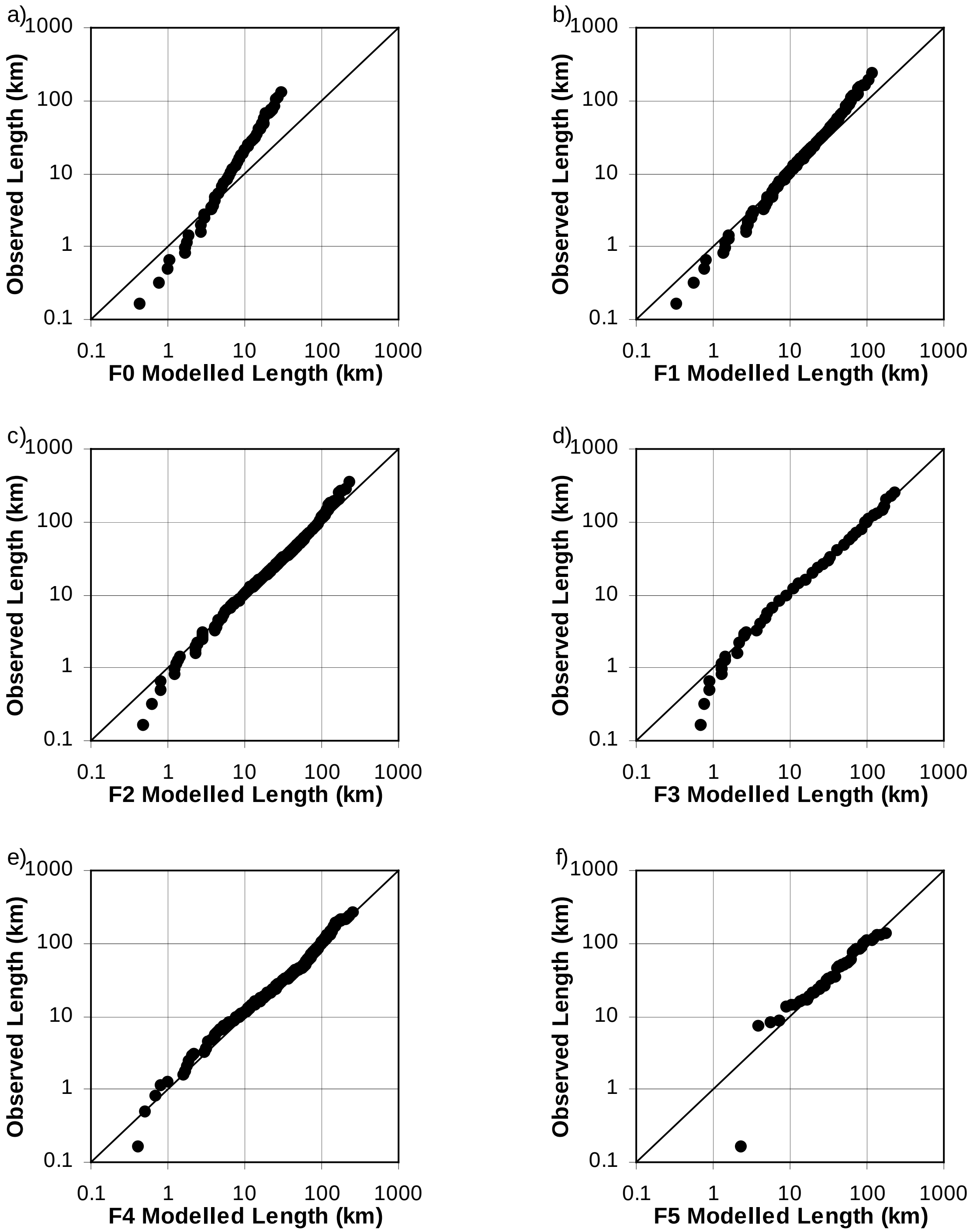
Fig. 8: Box and whisker plots for distribution of mean path length estimates based on random samples of four years. Top and bottom of box represents 75^{th} and 25^{th} percentiles. Top and bottom of whiskers represent 90^{th} and 10^{th} percentiles.

Fig. 9: Same as Fig. 8, except for path width estimates.

Fig. 10: Mean length of distribution of F3 tornadoes for overlapping 4-year periods. Bold horizontal lines represent 95% confidence intervals on long-term mean.

Fig. 11: Same as Fig. 10 except for mean width of F0 tornadoes.

Fig. 12: Same as Fig. 10 except for mean width of F3 tornadoes.



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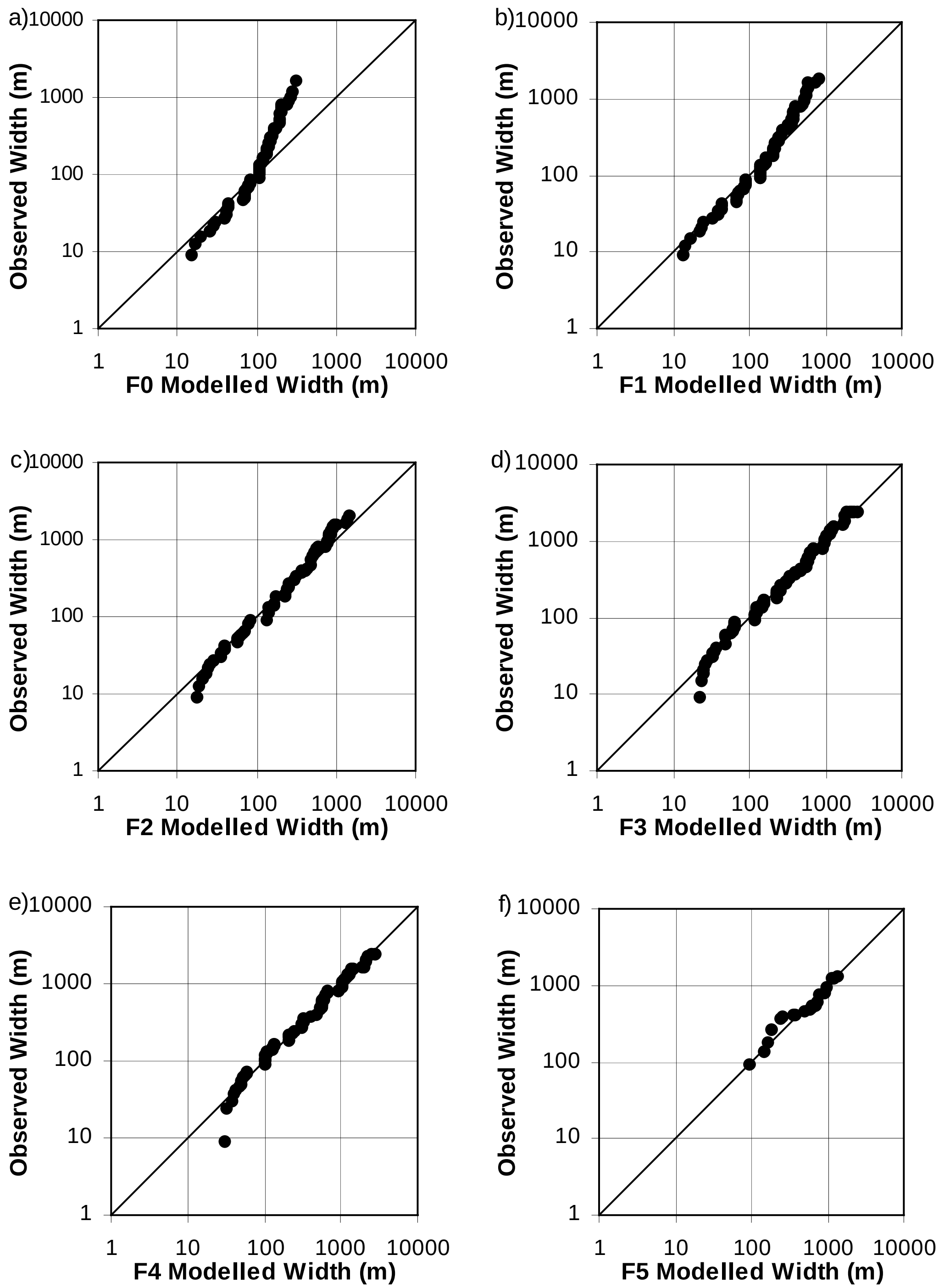

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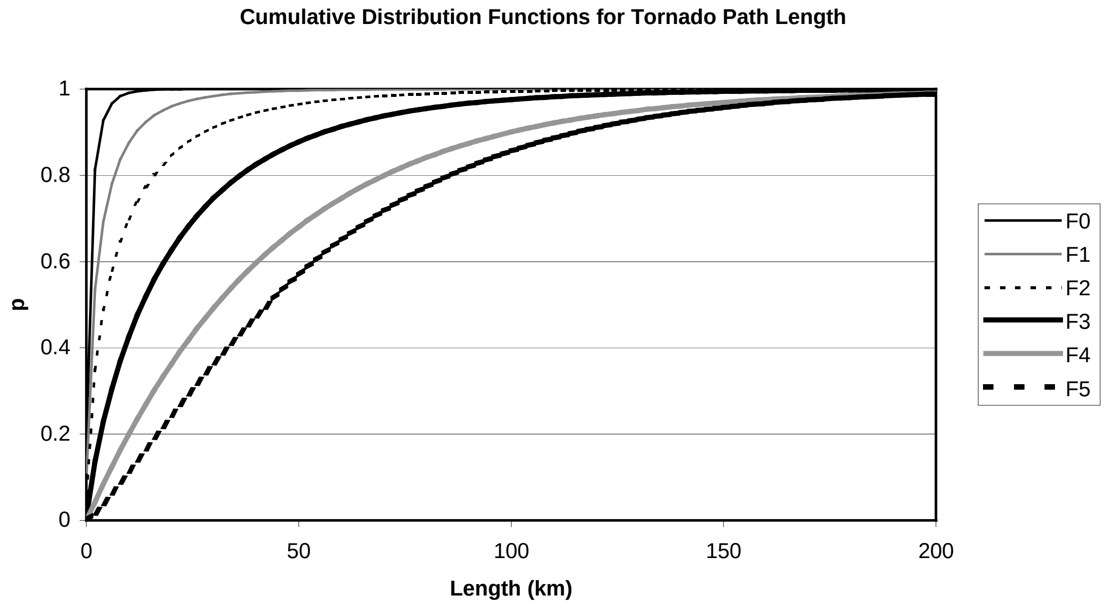

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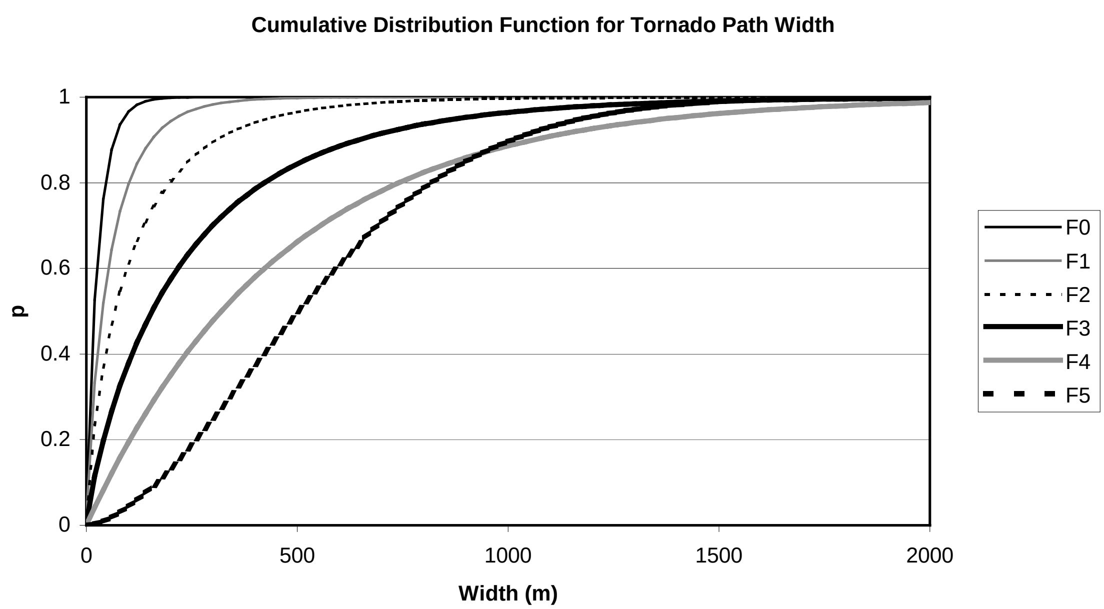

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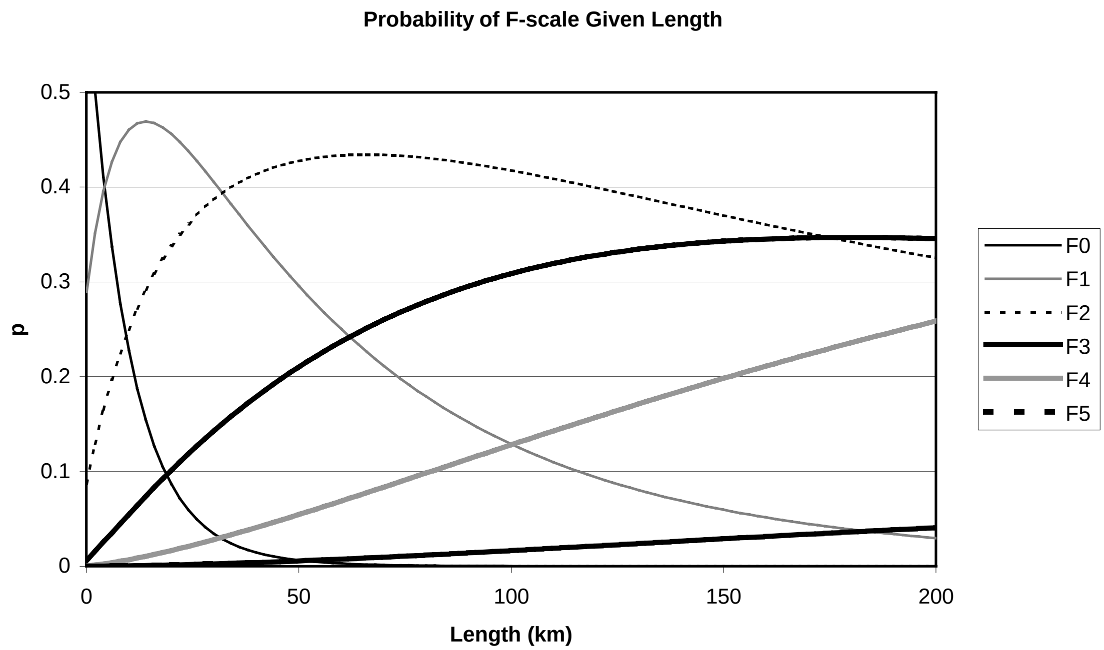

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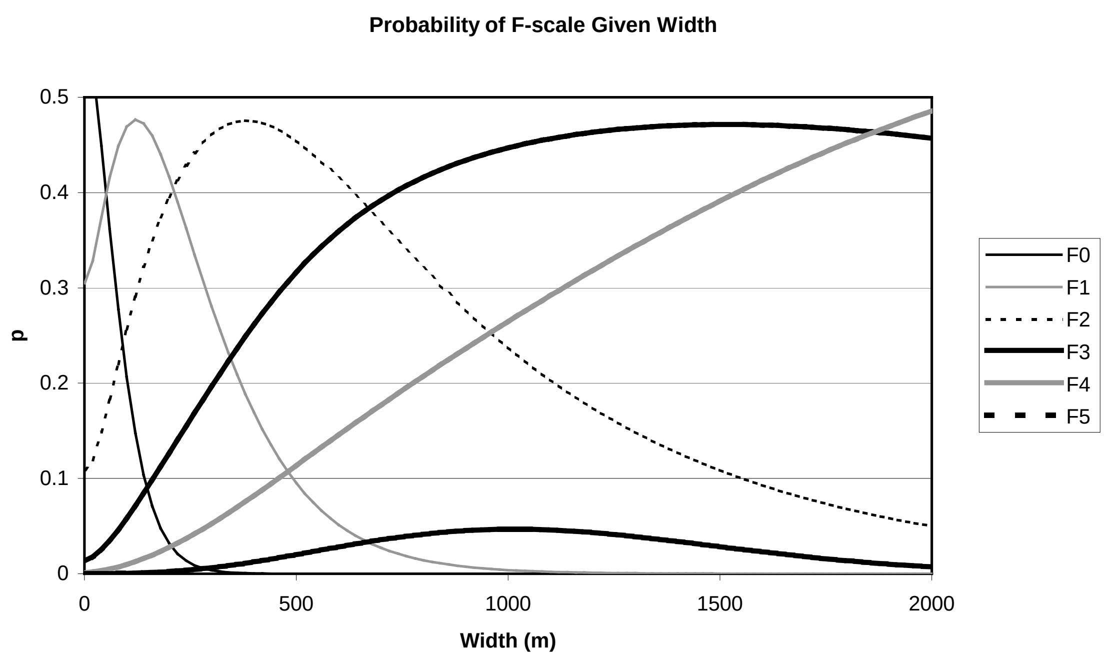

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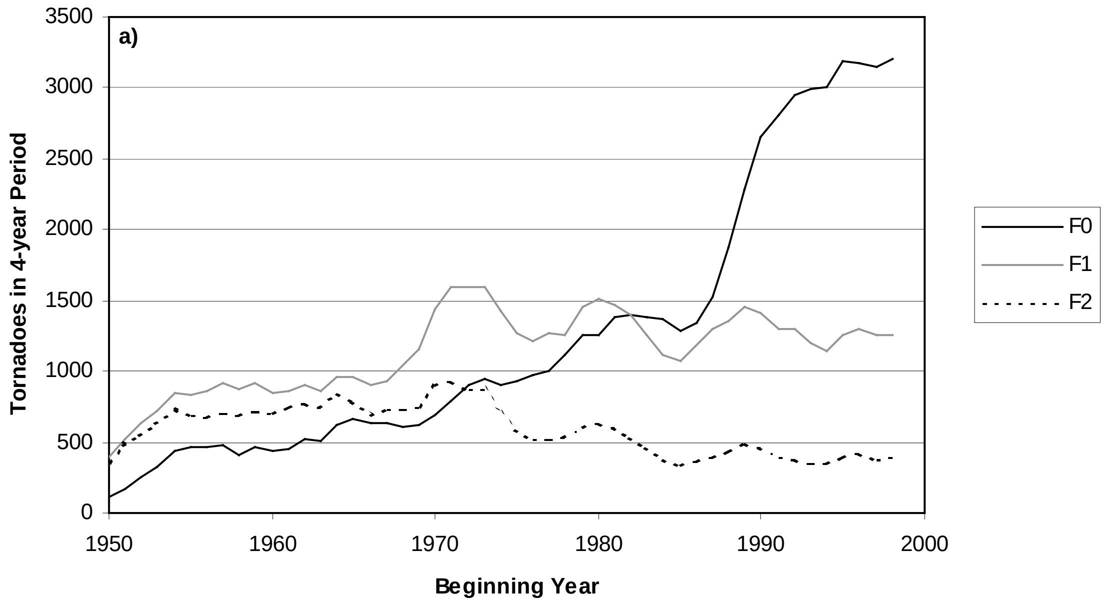

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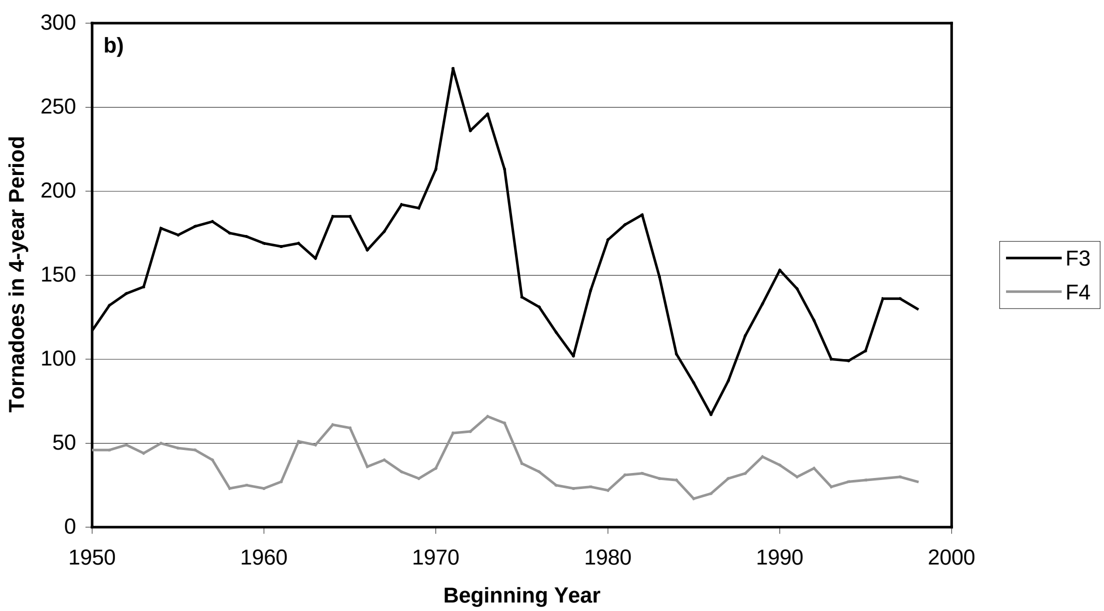

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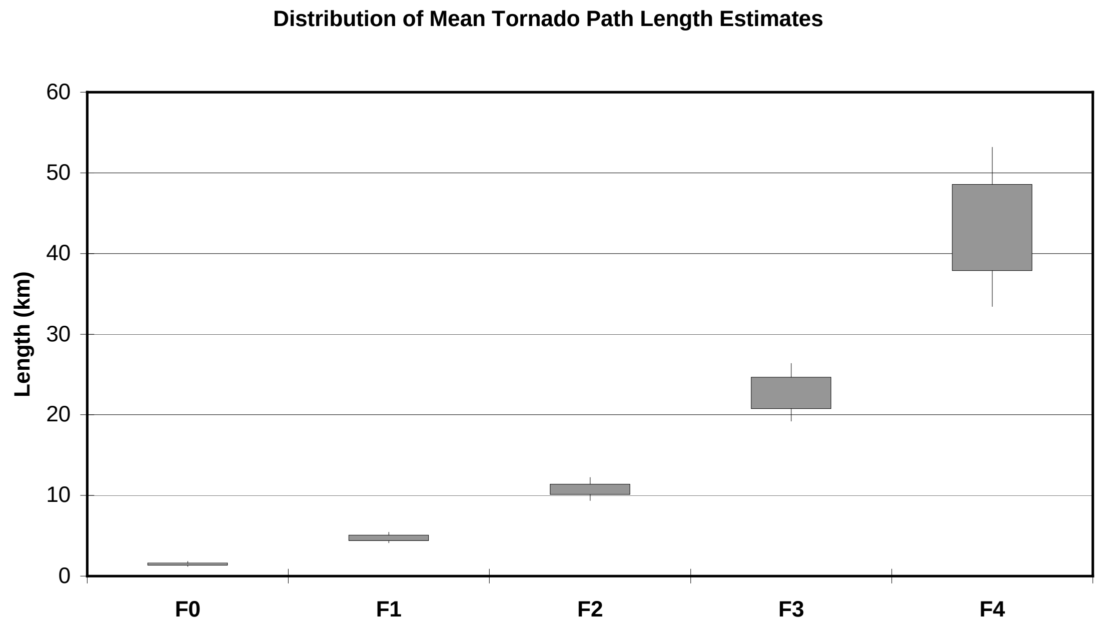

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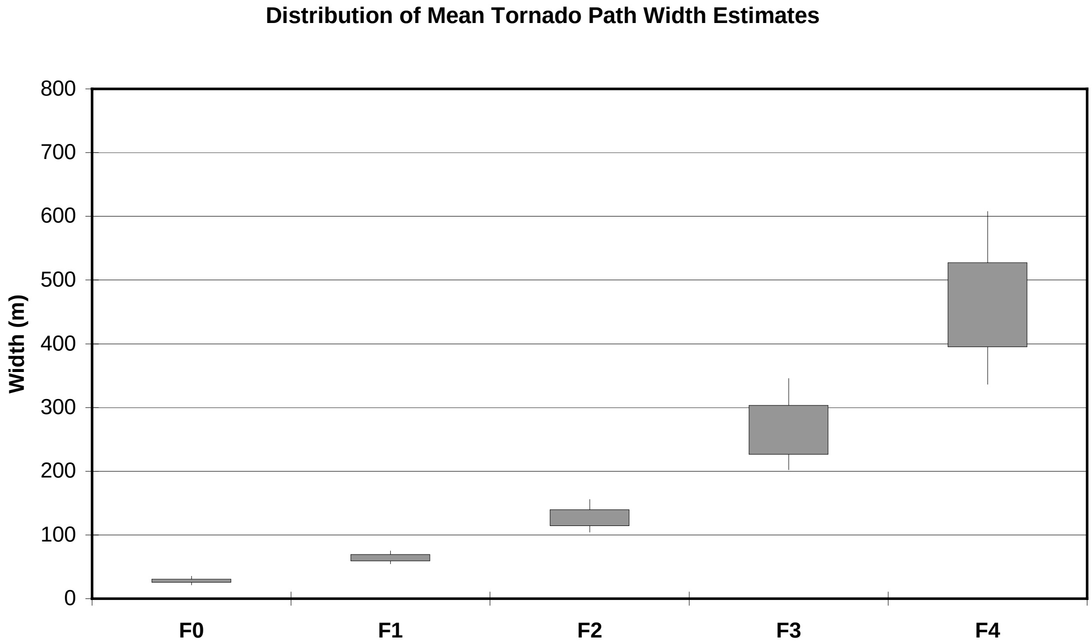
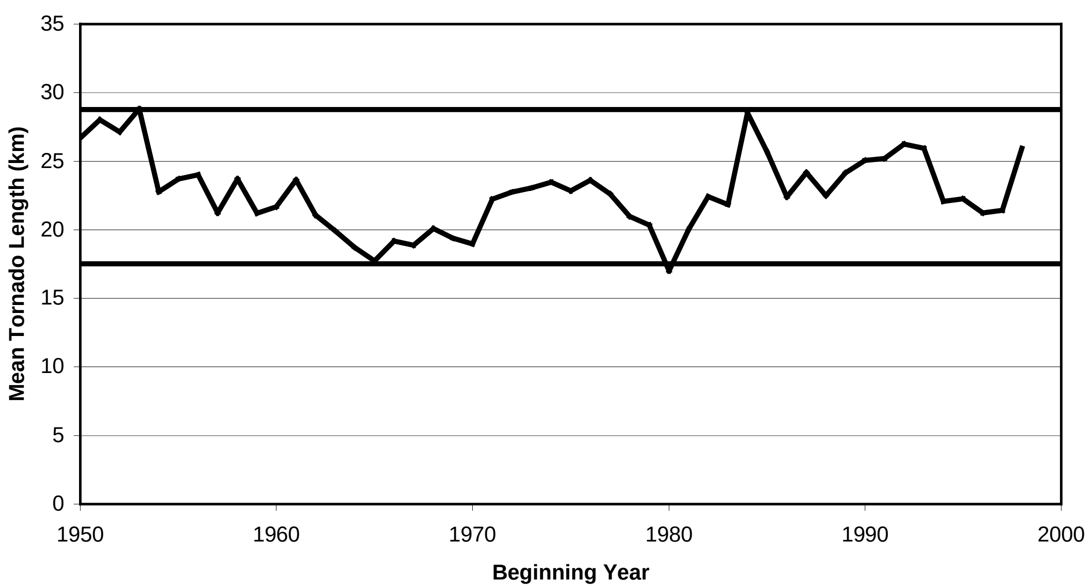


Fig. 9: Same as Fig. 8, except for path width estimates.



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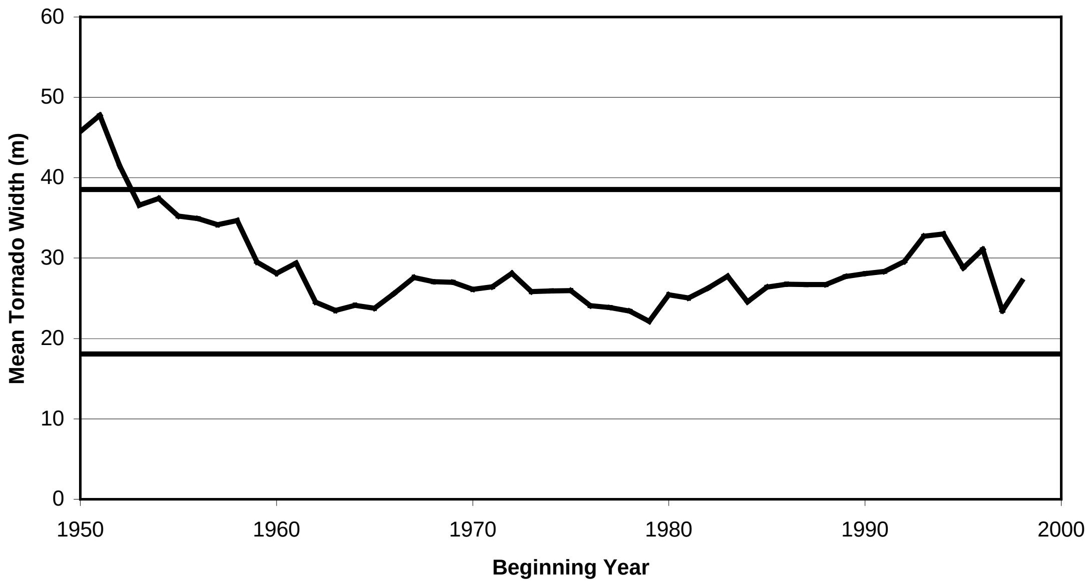
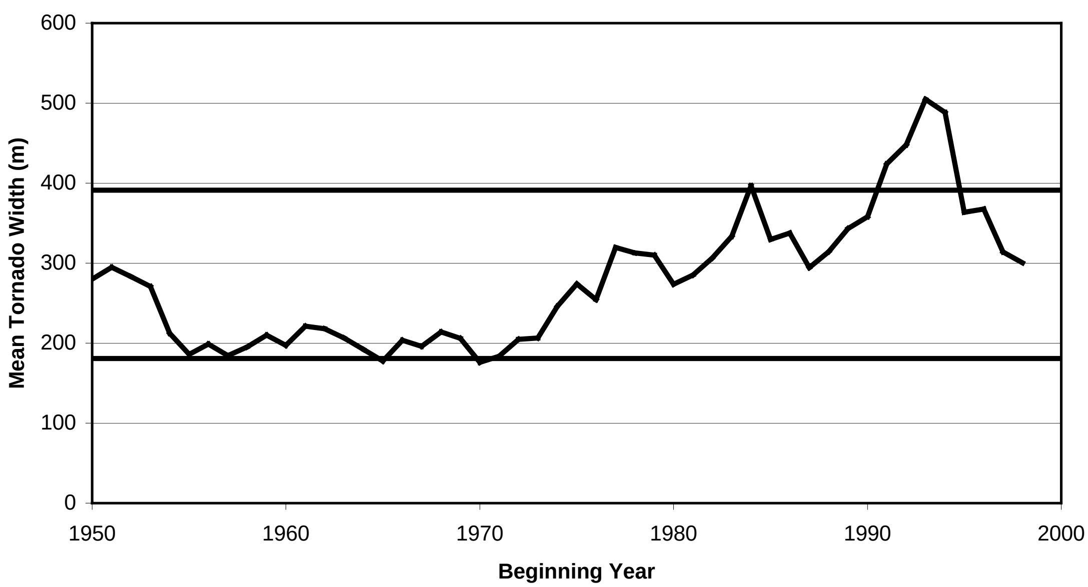


Fig. 11: Same as Fig. 10 except for mean width of F0 tornadoes.



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